CONSIDERATIONS ON THE STRESSES CONCENTRATION FACTOR

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Abstract: The paper analyzes stresses in an area of tension-type with geometric discontinuity concentrator. The geometric discontinuity is a hole located in a thin plate of finite dimension. The stresses and their variations were determined using the finite element method (FEM). Stresses values obtained with FEM, were used as basic data on the method of least squares to determine an analytical relationship, for calculating the coefficient of stress concentration. Relationship obtained by this method of calculation is compared with those in the technical literature.

Keywords: stress concentration factor, hole-tip stress distribution

1. INTRODUCTION

In the elements of a mechanical structure stresses and strains are produced, under the action of various loads acting on the structure. In some areas of the structure, where there are abrupt variations of the geometric form, significant increases of stress values occur. To determine stresses in the concentrator zone, aspects regarding the role of the geometric dimension of the concentrator on the highest values and on the way stresses vary in the shortened concentrator section have been analyzed.

2. COMPUTER COMPONENTS

The net stress concentration factor, in a concentrator area, as shown in Figure 1, is defined by:

$$k_{\sigma,n} = \frac{\sigma_{max}}{\sigma_n} \tag{1}$$

where: $k_{\sigma,\,n}$ is the net stress concentration factor, based on net stress, σ_{max} is the maximum stress on point *A*, of the concentrator, σ_n is the net stress of the section *A-B*.

The net stress of the section *A-B*, is considered to be uniformly distributed and will be determined with the relation:

$$\sigma_n = \frac{p \cdot B_o}{\left[B_o - 2R\right]} \tag{2}$$

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where: p is uniform stress in the strip, B_o is the width of the strip, R is the radius of the circular concentrator.

The stress concentration factor, k_σ in a concentrator area, as shown in Figure 1, is defined by:

$$k_\sigma = \frac{\sigma_{max}}{p} \tag{3}$$

The calculation of net stress variation was made according to the dimensional factor k_d, which is defined by the relation:

$$k_d = \frac{2 \cdot R}{B_o} \tag{4}$$

From the foregoing, we have:

$$k_{\sigma,n} = \frac{\sigma_{\max} \cdot (1 - k_d)}{p} = k_\sigma \cdot (1 - k_d) \tag{5}$$

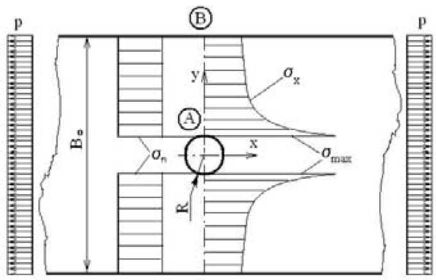

Fig. 1. The strip with stress concentrator.

2.1 Analytic and numeric calculation

The analytical solution is known for stress σ_x, that sets in motion section *A-B*, for the case of a circular hole in an infinite width plate which has a stress applied to a direction by a constant stress [1]. The stress σ_x in section *A-B* is determined with the relation:

$$\sigma_x = \left(1 + 0.5 \cdot \left(\frac{R}{y}\right)^2 + 1.5 \cdot \left(\frac{R}{y}\right)^4\right) \cdot p \tag{6}$$

where y is the coordinate point where stress is calculated.

For $y=R$, stress σ_x has the maximum value $\sigma_{x,\ max}=3p$. In this case the stress concentration factor is $k_\sigma=3$. Relation (6) can be used in the case in which, width B_o has high values in relation to the diameter of the hole so that $k_d \le 0.2$. If $k_d > 0.2$ calculation errors are higher.

To determine the calculation error when the dimensional factor is $k_d > 0.2$ the equivalent axial force of the weakened section of the strip was determined and was compared to the axial force of the whole width B_o. We will obtain in this case:

$$N_n = 2 \cdot \int_R^{\frac{B_o}{2}} \sigma_x dx \; ; \quad N = p \cdot B_o \tag{7}$$

where: N_n is the net equivalent axial force in the weakened section of the strip (Figure 1), N is the equivalent axial force in section B_o of strip.

Comparing the values of equivalent axial forces, corresponding to the two sections, we can determine the calculation errors on calculating the σ_x stress for the cases in which the k ratio varies in the range $k_d \in [0.2 \ldots 0.7]$.

$$\varepsilon = \frac{N - N_n}{N} 100 \tag{8}$$

Figure 2 presents the plot graphic variation of the calculation error ε.

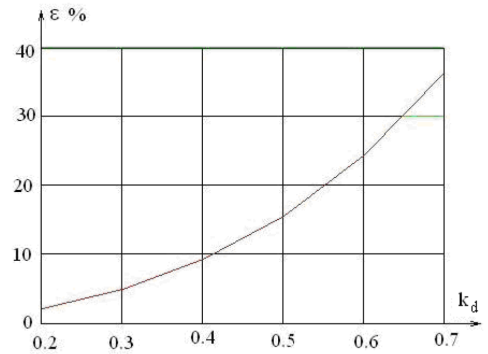

Fig. 2. Calculation error of stress σ_x.

The miscalculation is found to have significant values in the case $k_d > 0.4$. Thus, from a calculation error $\approx 10\%$ for $k_d = 0.4$, the error can reach 35% for $k_d = 0.7$.

To calculate the net stress concentration factor in papers [2 ÷ 5] the following relations have been suggested:

$$\text{(Heywood)} \quad k_{\sigma,He} = 2 + (1 - k_d)^3 \tag{9}$$

$$\text{(Roark)} \quad k_{\sigma,Ro} = 3 - 3.13 \cdot k + 3.66 \cdot k_d{}^2 - 1.53 \cdot k_d{}^3 \tag{10}$$

$$\text{(Howland)} \quad k_{\sigma,Ho} = 2 + 0.284 \cdot (1 - k_d) - 0.6 \cdot (1 - k_d)^2 + 1.32 \cdot (1 - k_d)^3 \tag{11}$$

Values obtained with relations (9), (10) and (11) were compared with a new relationship. The new relationship was determined with the method of least squares. Stresses values obtained with FEM, were used as basic data on the method of least squares.

The calculation model, of FEM was adopted taking into consideration the geometric and loading symmetries of the analyzed structure. Therefore only a quarter of the plate was considered for calculation; the shape and loading conditions are shown in Figure 3. Brick-type elements with ALGOR software was used.

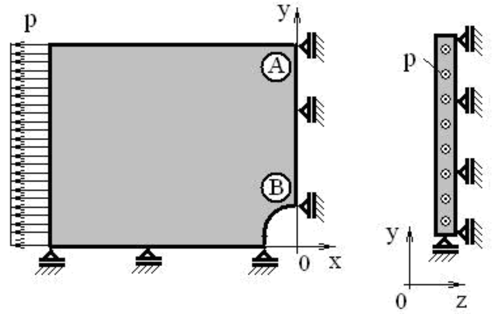

Fig. 3. The finite element analysis model.

For FEM model calculation the following values have been taken into account for k_d = 0.2; 0.3; 0.4; 0.5; 0.7; R=2 mm, and the strip thickness t=1 mm.

Figures 4 and 6, present stress σ_x and the way they vary on the flat strip section. Figures 5 and 7, present the variation of stress σ_x/p, and σ_n/p in section *A-B*.

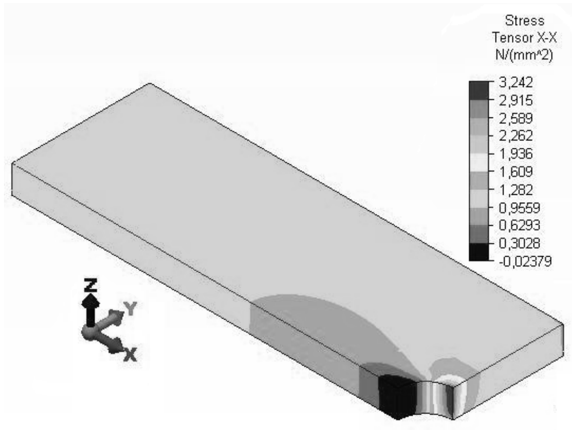

Fig. 4. Variation of stress σ_x for k_d=*0.2*.

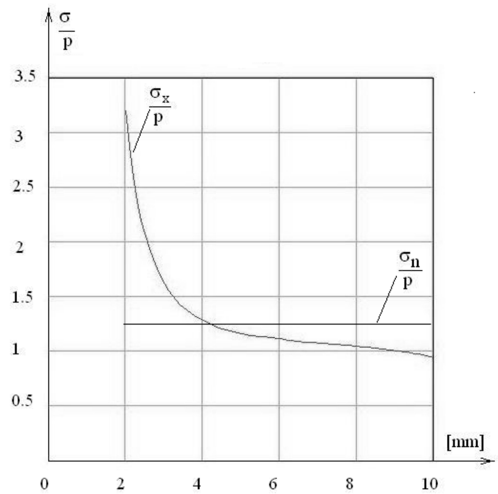

Fig. 5. Variation of stress σ_x and σ_n for k_d=*0.2*.

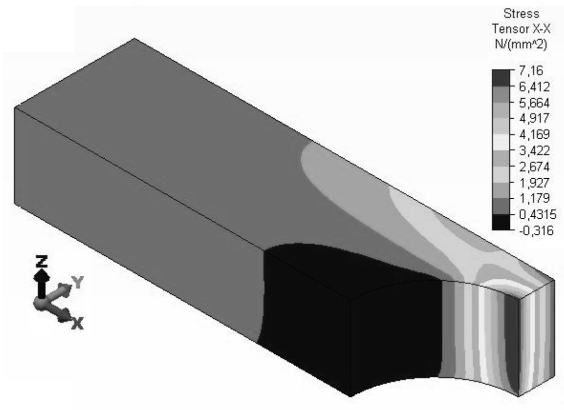

Fig. 6. Variation of stress σ_x for k_d=*0.7*.

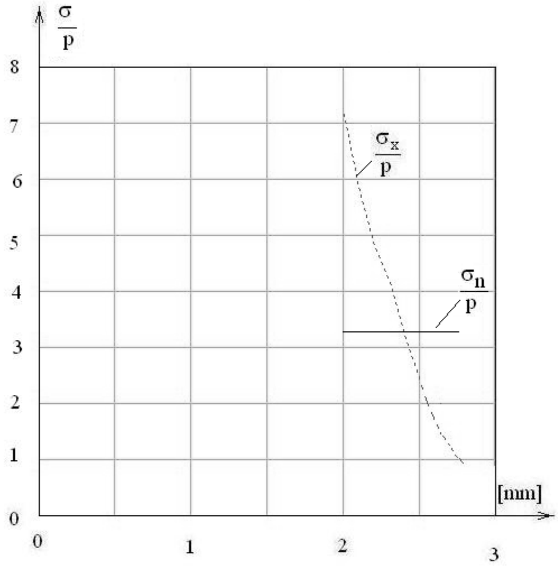

Fig. 7. Variation of stress σ_x and σ_n for $k_d=0.7$.

Method of least squares is considered for a lot of pairs of points (x_i, y_i) $i = 1 \ldots n$ data required to determine the expression of a function $ya_i = f(x_i)$ whose values at the points x_i to approximate values y_i as well.

Findings approximation method can be sum squares criterion function differences between initial values y_i and values, ya_i function calculated with the function $f(x)$ at points x_i.

We have:

$$E = \sum_{i=1}^{n} (y_i - ya_i)^2 \to \text{minimum} \tag{12}$$

For minimum error of equation (12) the conditions are:

$$\frac{\partial E}{\partial c_1} = 0; \ \frac{\partial E}{\partial c_2} = 0 : \ldots \frac{\partial E}{\partial c_n} = 0 \tag{13}$$

For *ya* function, I chose this relationship:

$$ya(x) = c_1 \cdot x^2 + c_2 \cdot x + c_3; \ (k_{\sigma,n}(k_d) = ya(x); \ \ x = k_d) \tag{14}$$

Applying the conditions (13) we obtain a system of three equations with three unknowns. We have:

$$\begin{bmatrix} \sum_1^n x_i^2 & \sum_1^n x_i^1 & n \\ \sum_1^n x_i^3 & \sum_1^n x_i^2 & \sum_1^n x_i^1 \\ \sum_1^n x_i^4 & \sum_1^n x_i^3 & \sum_1^n x_i^2 \end{bmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} \sum_1^n y_i \\ \sum_1^n x_i \cdot y_i \\ \sum_1^n x_i^2 \cdot y \end{pmatrix} \tag{15}$$

The value of the coefficients c_1, c_2 and c_3 where determined for the data presented in Table 1.

Table 1. Values of net stress concentration factor.

k	0.2	0.3	0.4	0.5	0.7
σ_{max} / σ_n	2.56	2.45	2.313	2.212	2.148

After solving systems of equations (15) will have:

$$k_{\sigma,n}(k_d) = 1.32 \cdot k_d^2 - 2.27 \cdot k_d + 3 \quad (16)$$

3. CONCLUSION

Figure 8 presents the diagrams of variation of the normal stress concentration factor $k_{\sigma,n}$ determined with relations (9), (10) and (11) and with approximation function.

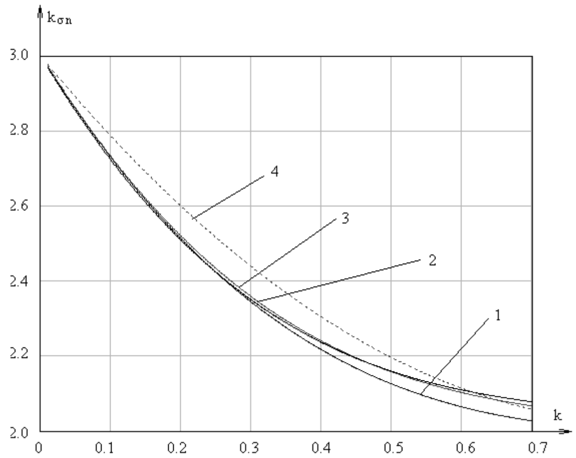

Fig. 8. Variation of stress concentration factors:
1 - Howland; 2 - Heywood; 3 - Roark's Formula; 4 - Approximation function.

Comparing the nominal coefficient values of stresses concentration $k_{\sigma,n}$ obtained with relation (16), with the coefficient values obtained with relations (9), (10) and (11) it is found that it is more than up to 4% for $k_d \in [0.3…0.4]$.

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