

## FREE TORSIONAL VIBRATIONS OF BIG DIAMETER DRILL STRING AND 10 $\frac{3}{4}$ " DRILL PIPE BREAKING

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**Abstract:** The purpose of the researches presented in this paper is to make evident the cracking and breaking causes of the 10 $\frac{3}{4}$ " drill pipes, used for the drilling of the mining shafts which have a diameter of 3.62 m. In this way, the free torsional vibrations of the 10 $\frac{3}{4}$ " drill string are studied by using the drill string model with non-uniform distributed mass and mass moments of inertia concentrated at the ends. Then, the electro-hydrostatic driving group inertia and the compressibility of the hydraulic medium in the hydrostatic transmission lines are taken into account. It is ascertained that the influence of the hydraulic medium compressibility is unfavorable from dynamic point of view because it decreases the natural angular frequency of torsional vibration in the working speed area of the drill string. The appearance of the resonance phenomenon in this working area is made evident by experimental researches and can be a reason regarding the breaking of the drill pipes assembled in the lower third of the drill string.

**Keywords:** drill pipe breaking, free torsional vibrations

### 1. INTRODUCTION

During the working of the 10 $\frac{3}{4}$ " drill string used for the drilling of the mining shafts having a diameter of 3.62m, by using the F320-3DH-M drilling rig, cracks and breakings of the drill pipes were occurred [1]. In this case, some theoretical studies and experimental researches [2-4] were carried out for making clear the reasons which made possible these phenomena. The present paper is situated in this frame studying the free torsional oscillations of the 10 $\frac{3}{4}$ " drill string.

The rotary system of the F320-3DH-M drilling rig is made (according to Figure 1) from two electro-hydrostatic driving groups (with axial piston units), rotary table (with two trains of cylindrical gears, each of them having three stages of speed decrease from the hydrostatic motor shaft to the rotary table rotor), driving ensemble (made of master bushing, kelly bushing, kelly and rotary swivel), drill string, drill collar, roller stabilizer and drilling bit. The drill string is made of drill pipes with air tubes, set up on the upper part, and drill pipes without air tubes. The air tubes are necessary to introduce compressed air inside the drill string at a certain depth for the drilling mud lifting, because the inverse circulation is used.

The dynamic calculation of the rotary system requires knowledge of the inertial and elastic values of its elements. This is way, firstly, a structural-dynamic analysis of this system was necessary, and on its basis, of a calculation of the inertia moments and of the torsion elastic constants of the component elements, and, at the same time, of compressibility modulus determination of the hydrostatic transmission hydraulic mineral oil. The rotary table, the drill string driving ensemble, itself the drill string, the drill collar, the stabilizer and the drilling bit are important elements from dynamic point of view of the system due to their masses in rotation motion, and,

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also, because the drill pipe elasticity. It is also important the hydraulic medium compressibility occurring in the hydrostatic transmission lines. It is ascertained that the drill collar, the stabilizer and the drilling bit can be considered as rigid elements, being characterized by concentrated high inertia moments. Alike, the driving ensemble and the rotary table motion elements may be accepted.

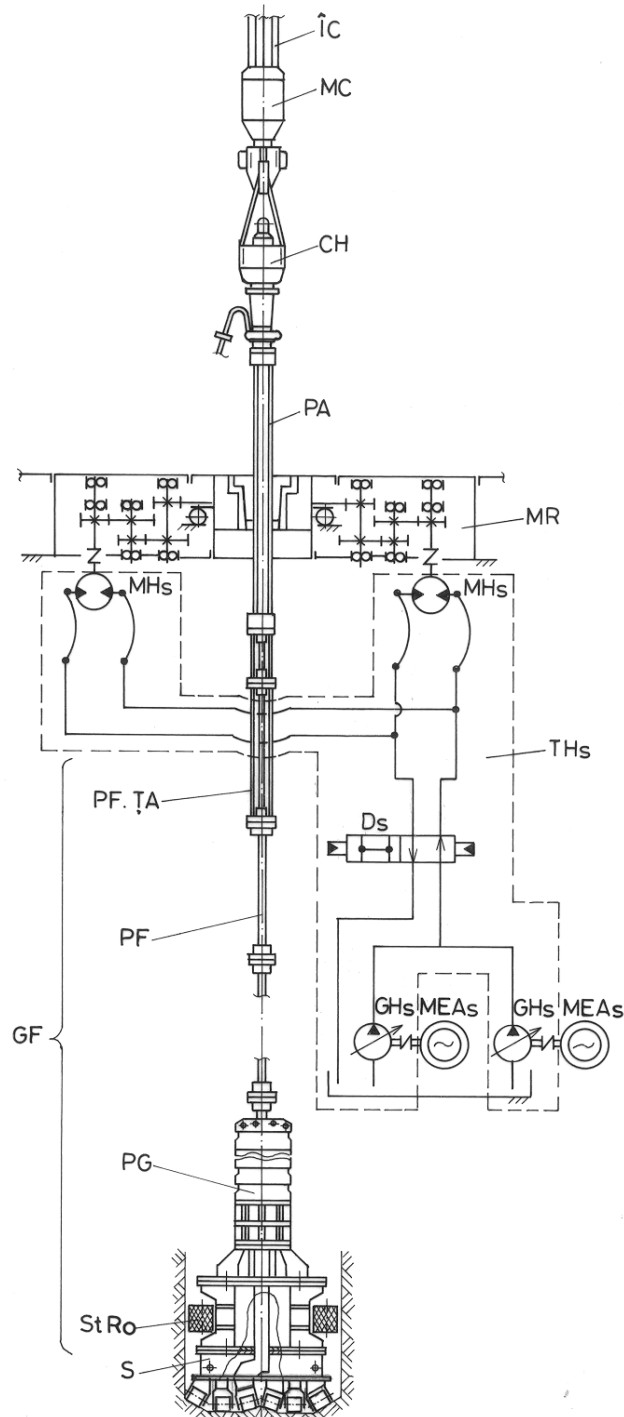


Fig. 1. Rotary system of the F320-3DH-M drilling rig: MEAs – asynchronous electric motor; THs – hydrostatic transmission; GHs – hydrostatic generator; Ds – distributor; MHs – hydrostatic motor; MR – rotary table; ÎC – the cable winding in the frame of the block-crown block machine; MC – hook-block; CH – swivel; PA – kelly; GF – drill string; PF.ÎA – drill pipes with air tubes; PF – drill pipes without air tubes; PG – drill collar; StRo – roller stabilizer; S – drilling bit.

In Figure 2 the drill pipe with air tubes construction is presented. The drill pipe jointing by means of flanges, two centering bolts for the torsion moment overtaking, and fixing screws introduces an important discontinuity in the drill pipe mass. In Figure 3 the bonding zone between two drill pipes is presented, making evident four elements having distinct geometrical characteristics, whose torsion elastic constants were calculated.

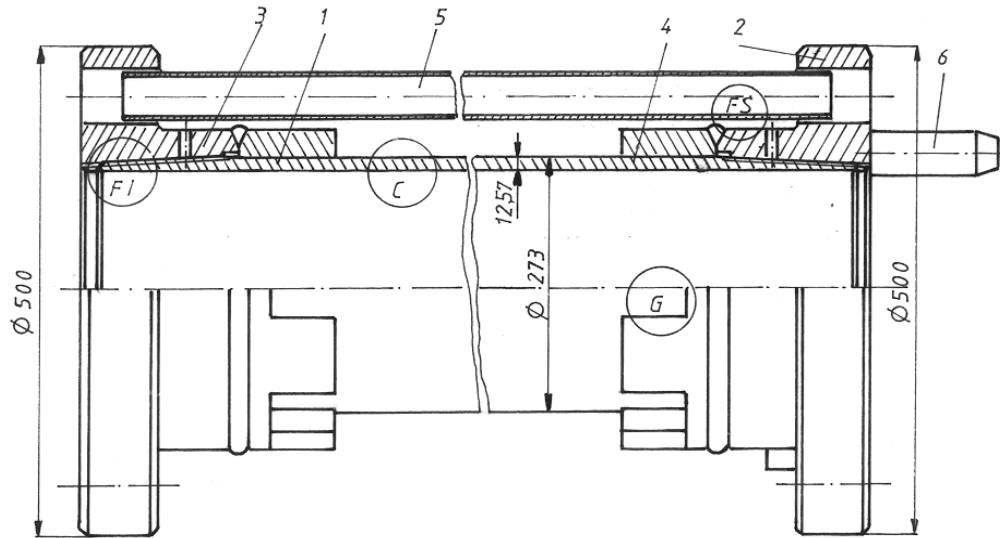


Fig. 2. Longitudinal section through a drill pipe with air tubes: 1 – drill pipe body; 2 – flange; 3 – flange neck; 4 – crenellated collar; 5 – air tube; 6 – centering bolt.

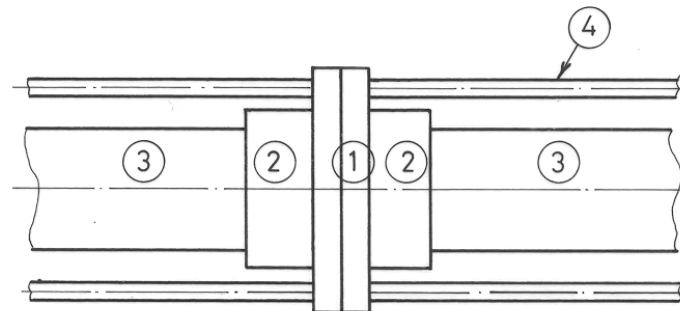


Fig. 3. The bonding zone between two drill pipes, with four elements having distinct geometrical characteristics: 1 – flange disc; 2 – flange neck and collar; 3 – drill pipe body; 4 – air tube.

During the drilling, torsional, longitudinal and flexural vibrating processes in the drill string are generated due to the three sources of dynamic excitations: the motor excitation source represented by the hydrostatic motors and the sources of reaction excitation or the “strong” reactions, due to the contact between the bit rollers and the rock under the action of the weight on bit and between the stabilizer rollers and the shaft wall. The drilling mud reaction on the drill string is characterized as a “soft” reaction, having a dissipative effect, and not exciting.

The increasing of the torsional oscillation amplitudes in certain working conditions (that is at a certain depth and for a certain weight on bit and rotation speed) ascertained during some experiments on the field, imposed a study of the free torsional oscillations. This study is further exposed.

## 2. FREE TORSIONAL VIBRATIONS OF THE DRILL STRING, CONSIDERED WITH AN UNIFORM DISTRIBUTED MASS AND MASS MOMENTS OF INERTIA CONCENTRATED AT THE ENDS

It is considered, in a first approximation, a drill string model characterized by (see Figure 4): an uniform distributed mass on the drill string length, with an equivalent polar moment of inertia of cross-section ( $I_{p,eq}$ ); mass moments of inertia concentrated at the ends ( $J_1$  and  $J_2$ ); the drill string ends (noted 1 and 2) being free.

The equivalent polar moment of inertia of cross-section is determined as well-balanced mean of the polar moments of inertia of the drill pipe three zones, as the following:

$$I_{peq} = \frac{l_p}{\frac{l_1}{I_{p.1}} + \frac{2 \cdot l_2}{I_{p.2}} + \frac{l_3}{I_{p.3}}} \quad (1)$$

where  $l_i$  and  $I_{p.i}$  are the lengths and the polar moments of inertia, respectively, of the zones  $i = 1, 2, 3$  and  $l_p$  is the length of a drill pipe,

$$l_p = 2 \cdot (l_1 + l_2) + l_3 \quad (2)$$

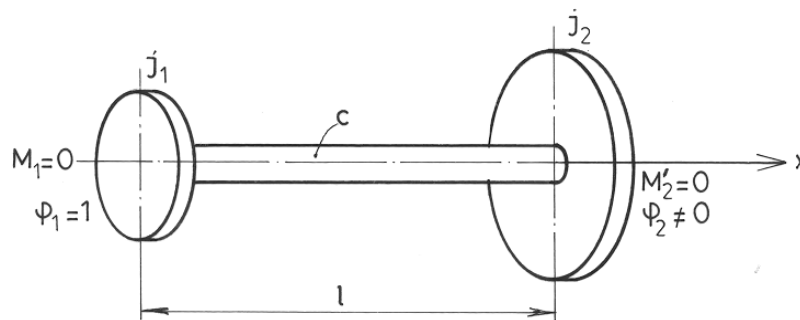


Fig. 4. The drill string model with uniform distributed mass and mass moments of inertia concentrated at the ends.

The torsion elastic constant of the drill pipe cross-section (the unitary torsion elastic constant) is determined by means of the following relationship:

$$C_1 = G \cdot I_{peq} \quad (3)$$

where  $G$  represents the shear modulus of the steel.

At the ends of the drill string the concentrated mass moments of inertia are considered in the shape of some flywheels, namely: at the upper end (1), the mass moment of inertia reduced at the Kelly, noted with  $J_1$ , of the elements in rotation motion contained between the hydrostatic motors, inclusively these ones, and the Kelly, together with the swivel; at the lower end (2), the mass moment of inertia of the drill collar, of the roller stabilizer and of the drilling bit, noted with  $J_2$ .

On the basis of the resulted mathematic model, taking into account the boundary conditions [5], the following equation of the natural angular frequency ( $p$ ):

$$\operatorname{tg}\left(\frac{p}{c_t} \cdot l\right) = \frac{\frac{p}{c_t} \cdot l}{\frac{c_t^2 \cdot J_1 \cdot J_2}{C_1 \cdot l \cdot (J_1 + J_2)} \cdot \left(\frac{p}{c_t} \cdot l\right)^2 - \frac{C_1 \cdot l}{c_t^2 \cdot (J_1 + J_2)}} \quad (4)$$

where  $c_t$  is the velocity of torsional impulses through the drill pipe material and  $l$  – the drill string length,

$$l = n_p \cdot l_p, \quad (5)$$

$n_p$  being the number of drill pipes.

For  $l_p = 6.94$  m,  $n_p = 45$ ,  $I_{p.eq} = 1.81099 \cdot 10^{-4}$  m<sup>4</sup>,  $C_1 = 1.4669037 \cdot 10^7$  N·m<sup>2</sup>,  $J_1 = 12173$  kg·m<sup>2</sup>,  $J_2 = 91000$  kg·m<sup>2</sup>, by numerical solving of the transcendental equation (4), the following measures of the natural angular frequency in case of 10<sup>3</sup>/<sub>4</sub>" drill string is obtained:  $p_1 = 2.098$  s<sup>-1</sup>;  $p_2 = 33.218$  s<sup>-1</sup>;  $p_3 = 66.156$  s<sup>-1</sup>;  $p_k = 10.529 \cdot (k-1) \cdot \pi$  s<sup>-1</sup>,  $k \in \mathbf{N} - \{1, 2\}$ .

### 3. FREE TORSIONAL VIBRATIONS OF THE DRILL STRING MODEL WITH NON-UNIFORM DISTRIBUTED MASS AND MASS MOMENTS OF INERTIA CONCENTRATED AT THE ENDS

The graphic model of the drill string with non-uniform distributed mass and mass moments of inertia concentrated at the ends is presented in Figure 5.

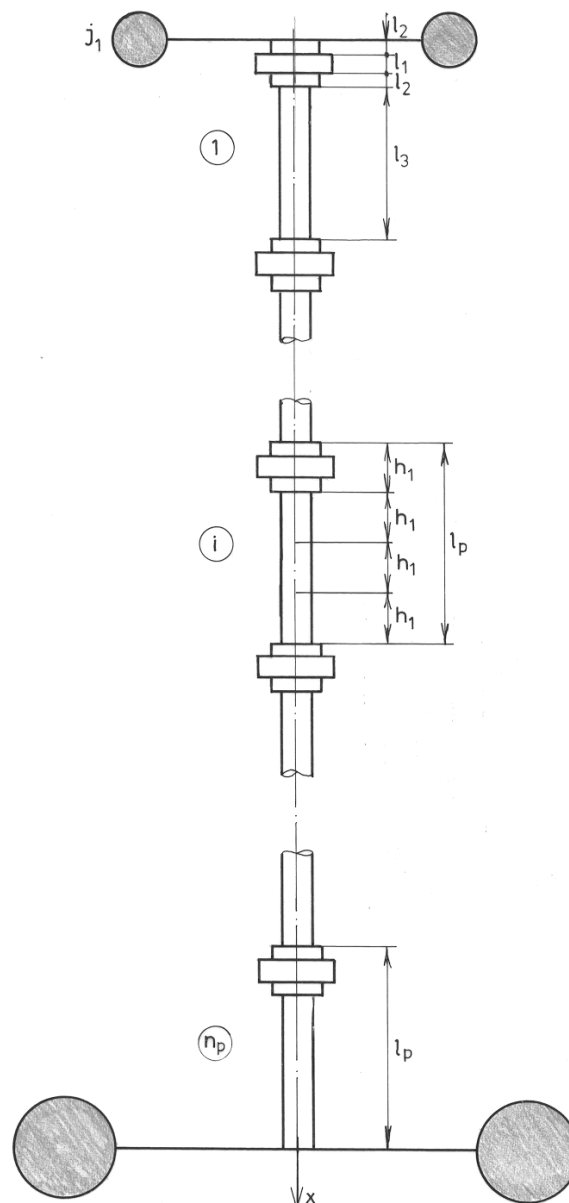


Fig. 5. The graphic model of the 10<sup>3</sup>/<sub>4</sub>" drill string with non-uniform distributed mass and mass moments of inertia concentrated at the ends.

In this case, the mathematical model which describes the free torsional vibrations of the drill string is expressed in the following equation system:

$$-p^2 \cdot J_1 \cdot \varphi_1 - M'_1 = 0 \quad (6)$$

$$\varphi_j = \frac{1}{\alpha} \cdot \frac{1}{C_{ij}} \cdot M_i \cdot \sin(\alpha \cdot l_{ij}) + \varphi_i \cdot \cos(\alpha \cdot l_{ij}) \quad (7)$$

$$M_j = M_i \cdot \cos(\alpha \cdot l_{ij}) - \alpha \cdot C_{ij} \cdot \varphi_i \cdot \sin(\alpha \cdot l_{ij}) \quad (8)$$

$$-p^2 \cdot J_2 \cdot \varphi_2 + M_2 = 0 \quad (9)$$

where  $l_{ij}$  is the length of the  $i$ - $j$  space (with the ends  $i$  and  $j$ );  $C_{ij}$  is the torsional stiffness coefficient of the  $i$ - $j$  space cross-section (unitary torsion elastic constant of the  $i$ - $j$  space);  $M_k$  are the torque in section  $k = i, j$ ;  $\varphi_k$  is the angular displacement of the section  $k = i, j$  (see Figure 5).

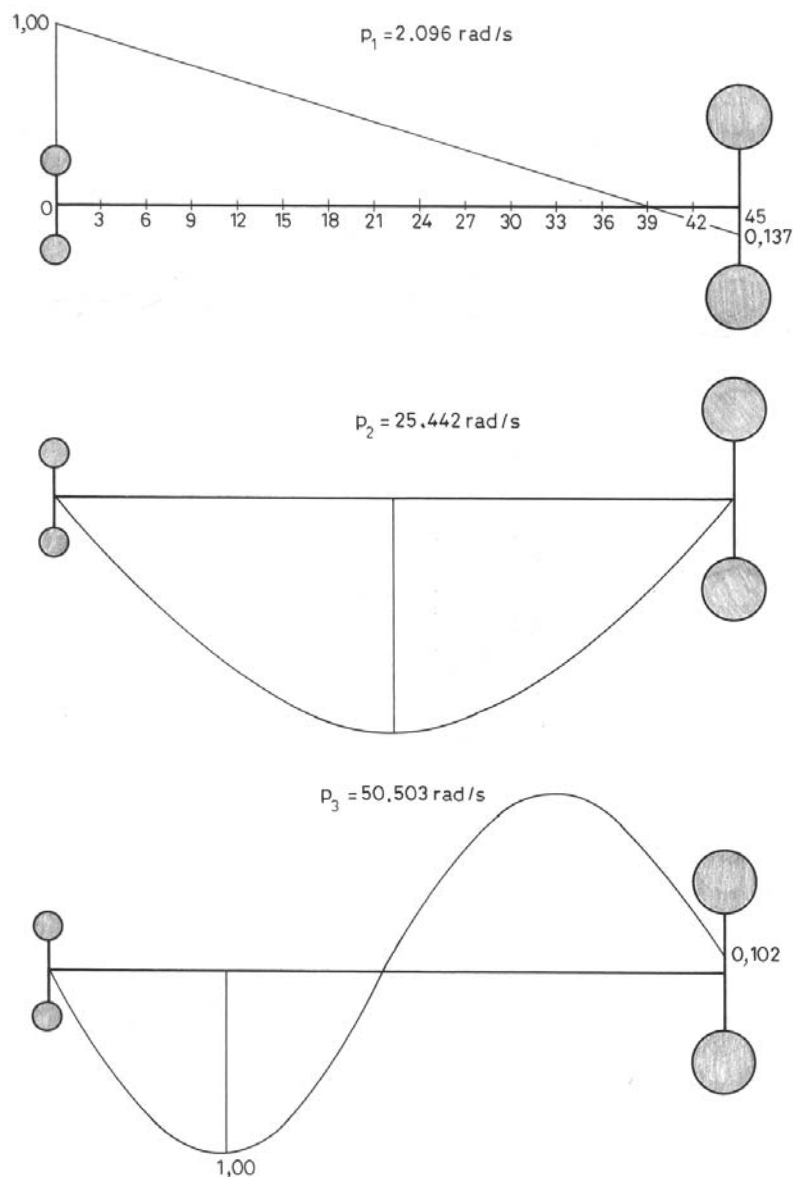


Fig. 6. The first three natural modes of torsional vibration of the 10 $\frac{3}{4}$ " drill string, with non-uniform distributed mass and mass moments of inertia concentrated at the ends.

The calculation stages are the followings: 1) an initial measure of natural angular frequency,  $p$ , is selected; 2) the initial parameters for the upper end of the drill string,  $M_0 = 0$ ,  $\varphi_0 = 1$ , it is considered; 3) the recurrence relations (7) and (8) are applied, and  $M$  and  $\varphi$  are calculated; 4) it is numerical solved equation  $M = f(p)$  for determination the natural angular frequencies; 5) by using the relations (7) and (8), the angular displacement  $\varphi$  corresponding to the determined natural angular frequency is calculated, and the natural modes of torsional vibration are obtained.

On the basis of the above presented algorithm, a program in MATLAB language was realized, and the following measures of the first six natural angular frequencies were obtained:  $p_1 = 2.096 \text{ s}^{-1}$ ;  $p_2 = 25.422 \text{ s}^{-1}$ ;  $p_3 = 50.505 \text{ s}^{-1}$ ;  $p_4 = 75.487 \text{ s}^{-1}$ ;  $p_5 = 100.257 \text{ s}^{-1}$  and  $p_6 = 124.727 \text{ s}^{-1}$ , finding that these measures are lower than in the case of the drill string model with uniform distributed mass. The natural modes of torsional vibration for the first three natural angular frequencies are represented in Figure 6.

#### 4. INFLUENCE OF INERTIA AND ELASTICITY OF THE ELECTRO-HYDROSTATIC DRIVING GROUP ON THE FUNDAMENTAL NATURAL ANGULAR FREQUENCY OF THE TORSIONAL VIBRATIONS IN CASE OF 10¾" DRILL STRING

To estimate the influence of inertia and elasticity of the electro-hydrostatic driving group elements on the fundamental natural angular frequency in case of torsional vibrations of the 10¾" drill string it is necessary to reduce the mass and the elastic constant of these ensemble elements at the kelly.

It is taken into consideration only the hydraulic medium compressibility occurring in the flow lines of the hydrostatic transmission, by neglecting the elasticity of the elastic clutches with bolts between the electric motor and the pump, and between the hydrostatic motor and the driving shaft of the rotary table. In this way, on the basis of the potential energy equivalence of the real system and its model, the elastic constant of the ensemble hydraulic medium-flow line, reduced at the kelly, will result:

$$C_{red} = k \cdot \left( \frac{\Delta x}{\Delta \varphi} \right)^2 \quad (10)$$

where  $k$  is the elastic constant of the ensemble hydraulic medium-flow line;  $\Delta \varphi$  is the angular strain of the kelly, corresponding to the elastic strain  $\Delta x$  of the motive fluid occurring in the flow line.

Taking into account the occurring of the individual high pressure lines (IHPL) where each of the pumps will discharge, and of the main high pressure line (MHPL),  $k$  is calculated in the following way:

$$\frac{1}{k} = \frac{1}{k_{\Sigma IHPL}} + \frac{1}{k_{MHPL}} \quad (11)$$

$$k_{\Sigma IHPL} = \sum_{j=1}^N k_{IHPL.j} \quad (12)$$

where  $N$  is the number of the hydrostatic generator groups being set in working.

The elastic constant of a line, individual or main, noted with  $k_C$ , is determined by means of formula:

$$k_C = \frac{E_{eq} \cdot A_C}{L_C} \quad (13)$$

$A_C$  being the line flowing section area;  $L_C$  is the line length;  $E_{eq}$  is the equivalent compressibility modulus [2],  $E_{eq} = 1312.5 \text{ MPa}$ .

By means of condition:

$$\Delta\varphi \cdot V_{1N} = \Delta x \cdot A_{Ceq} \quad (14)$$

the expression of the reduced elastic constant becomes:

$$C_{red} = k \cdot \left( \frac{V_{1N}}{A_{Ceq}} \right)^2 \quad (15)$$

where  $V_{1N}$  is the liquid volume discharged by the  $N$  hydrostatic generators in case of a complete rotation, and  $A_{Ceq}$  is the equivalent flowing section area.  $A_{Ceq}$  is determined by the following formula:

$$A_{Ceq} = \frac{\sum_{(k)} L_{Ck} \cdot A_{Ck}}{\sum_{(k)} L_{Ck}} \quad (16)$$

where  $A_{Ck}$  is the line following section area having order  $k$ ;  $L_{Ck}$  – the  $k$  line length. For the hydrostatic transmission of the F320-3DH-M drilling rig,  $C_{red} = 5\,802.53$  Nm is obtained.

The elastic constant of the drill string, calculated by means of expression:

$$C = \frac{G \cdot I_{peq}}{l} \quad (17)$$

has the measure 47939 Nm.

It is found that the hydrostatic transmission elasticity is about 8.3 times higher than that of the drill string, this means that the hydrostatic transmission, by the hydraulic medium compressibility and the line elasticity, carries out the function of a dynamic absorber.

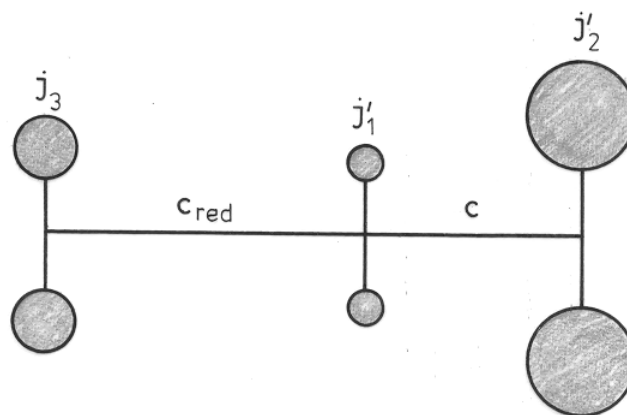


Fig. 7. The rotary system model of the F320-3DH-M drilling rig having three concentrated masses and two elastic bonds.

For the rotary system, the model presented in Figure 7 is adopted. This model is made from three concentrated masses, having the mass moments of inertia  $J'_1$ ,  $J'_2$  and  $J_3$ , with two elastic bonds with the constants  $C$  and  $C_{red}$ .  $J_3$  is the mass moment of inertia of the elements being in rotation motion of the electric motors and hydrostatic generators reduced at the kelly. It resulted:  $J_3 = 28426 \text{ kg} \cdot \text{m}^2$ . The mass moments of inertia  $J'_1$  and  $J'_2$  are determined from conditions which express the equivalence of the sub-system represented in Figure 7 by  $J'_1$ ,  $J'_2$  and the constant  $C$  with the system whose model is presented in Figure 5. In this way, it obtains:



$$J'_1 = \frac{J_1 + J_2}{2} - \sqrt{\frac{(J_1 + J_2)^2}{4} - C \cdot \frac{J_1 + J_2}{p_1^2}} \quad (18)$$

$$J'_2 = \frac{J_1 + J_2}{2} + \sqrt{\frac{(J_1 + J_2)^2}{4} - C \cdot \frac{J_1 + J_2}{p_1^2}} \quad (19)$$

By using the transfer matrix method for the purpose of writing the relation between the state vectors from the system ends in Figure 7, the following biquadrate equation of the natural angular frequency is obtained:

$$p^4 - 2 \cdot A \cdot p^2 + B = 0 \quad (20)$$

where the constants  $A$  and  $B$  have the shape:

$$A = \frac{J_3 + J'_1}{2 \cdot J_3 \cdot J'_1} \cdot C_{red} + \frac{J'_1 + J'_2}{2 \cdot J'_1 \cdot J'_2} \cdot C \quad (21)$$

$$B = \frac{(J'_1 + J'_2 + J_3) \cdot C \cdot C_{red}}{J'_1 \cdot J'_2 \cdot J_3} \quad (22)$$

By solving the respective calculations, the following measures of the first two natural angular frequencies will result:  $p_1 = 0.48673 \text{ s}^{-1}$ ;  $p_2 = 2.19732 \text{ s}^{-1}$ .

## 5. CONCLUSIONS

In this paper, the free torsional oscillations of the 10 $\frac{3}{4}$ " drill string are studied. This drill string is used for the high diameter drilling of 3.62 m, by using the F320-3DH-M drilling rig. The fact which imposed this study was the increasing of the torsional oscillation amplitude of this drill string in certain conditions, made evident by the experimental recordings carried out on the field.

On the basis of a structural-dynamic analysis of the F320-3DH-M drilling rig rotary system, of the 10 $\frac{3}{4}$ " drill string, respectively, it is ascertained that there are elements in rotation motion, with big mass, which can be considered as rigid elements, with concentrated mass, as the driving ensemble elements (the drill collar, the stabilizer and the drilling bit), and the much more elastic elements: the drill pipes and the hydraulic medium of the hydrostatic transmission lines.

Firstly, the free torsional vibrations of the drill string model are studied, with moments of inertia concentrated at the ends and uniform distributed mass, taking into account the structural discontinuity represented by drill pipe joining with flanges, centering bolts and fastening screws, by using the calculation of an equivalent polar moment of inertia, and of the corresponding torsional unitary elastic constant.

The drill string model having non-uniform distributed mass and mass moments of inertia concentrated at the ends make evident lower measures of the natural angular frequency than in case of drill string model with uniform distributed mass. The representation of the natural fundamental mode of vibration shows that its node appears in the last third of the drill string that is there where cracks and breakings of the drill pipes are generated.

Taking into consideration the electric motor and hydraulic generator inertia, and also the hydraulic medium compressibility occurring in lines in the frame of the model with three concentrated masses, adopted for the rotary system, may be found that the hydrostatic transmission influence is unfavorable from dynamic point of view, because decreases the natural fundamental frequency of the torsional oscillations in the working zone of the drill string angular speed, [0.471; 1.257]·rad/s. As a result, the drill string running with an angular speed out of the area [0.365; 0.608]·rad/s is imposed in order to avoid the resonance phenomenon.

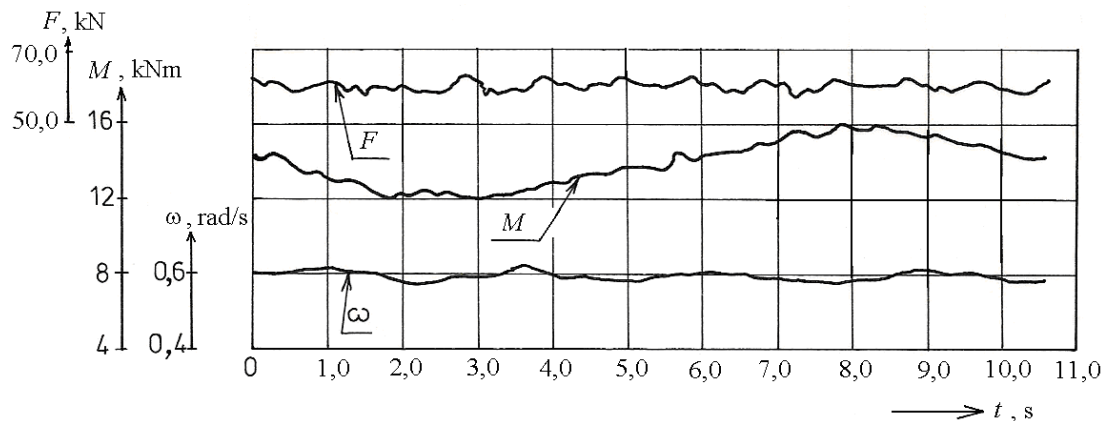


Fig. 8. The torque variation, which loads the upper drill pipe of the drill string, in case of a complete rotation, for the average weight on bit ( $F$ ) of 60 kN and the average angular speed ( $\omega$ ) of 0.585 rad/s.

The resonance phenomenon appearance in running zone of the 10 $\frac{3}{4}$ " drill string is made evident by the experimental researches occurred in Buştenari oil field by using the F320-3DH-M drilling rig. For example, in Figure 8 the recording of the torque, which loads the upper pipe of the drill string during the drilling, with an average weight on bit of about 60 kN, and the average angular speed of 0.585 rad/s, is presented. It is may be observed that the torque has a pulsating character, being approximately describes by a sinusoidal variation, having the shape:

$$M \cong 14 - 1.99 \cdot \sin(0.585 \cdot t) \quad (23)$$

where the angular frequency of the disturbing torque is 0.585 rad/s.

Also, the experimental researches carried out in the Stand of Research of the Non-Stationary Hydro-mechanical Processes [2, 6], which simulates, on a real scale, the rotary system of the F320-3DH-M drilling rig, reduced to the hydrostatic transmission, demonstrated the appearance of the knocking phenomenon (see Figure 9), that is some forced oscillations with angular frequency close to the natural angular frequency of the system, for the hydrostatic motor running to the rotation speed of 540 rot/min (that means the drill string angular speed of 0.568 rad/s, taking into account the gear ratio of the rotary table), with a braking moment of the hydraulic brake corresponding to the torque yielding during the drilling and flywheel with an mass moment of inertia reduced to the hydrostatic motor shaft corresponding to that of the drill string during the experiment in the oil field.

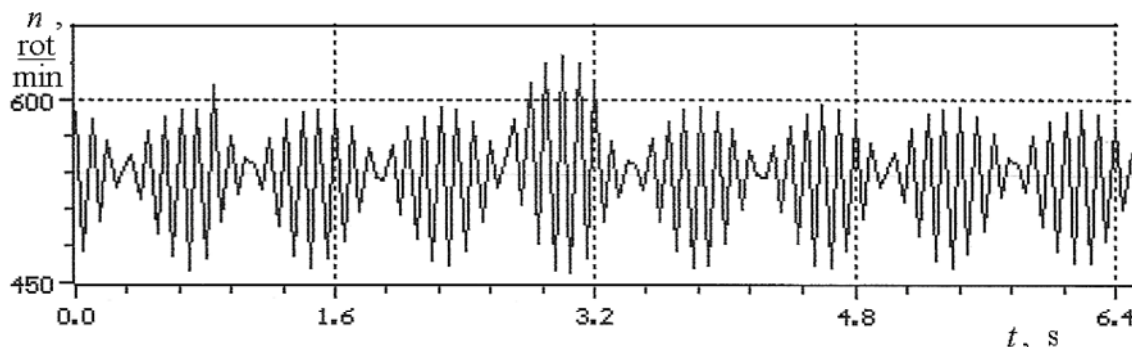


Fig. 9. Variation of the hydrostatic motor rotation speed ( $n$ ), with appearance of the knocking phenomenon, for the average rotation speed of 540 rot/min, in the frame of the Stand of Research of the Non-Stationary Hydro-mechanical Processes [2].

The study of the free flexural oscillations, in accordance with [2], made evident that the natural angular frequencies of orders 2, 3 and 4 may be found in the running area of the drill string, and the natural modes have nodes in the last third of the drill string, where, as it was above mentioned, drill pipes breaking had been occurred. Thus, it may be appreciated that the drill string running in resonance conditions of the torsional

vibrations, and especially flexural ones, contributed to initiation and propagation of the cracks until the breaking stage of the drill pipes, which were situated in the lower zone of the drill string.

## REFERENCES

- [1] Parepa, S., Analysis of the fracturing process of some drill pipes used to the high diameter drilling, *Lucrările celui de-Al XIV-lea Simpozion Internațional de Mecanica Ruperii*, Brașov, România, 2008, p. 51-58.
- [2] Parepa, S., *Studiul sistemului de rotație al instalațiilor pentru forajul sondelor de diametre mari*. Teză de doctorat, Universitatea din Petroșani, 2007.
- [3] Parepa, S., Variability of the loading in case of drill string used for high diameter well drilling, *Lucrările celui de-al XV-lea Simpozion Național de Mecanica Ruperii*, Sibiu, România, 2009, p. 81-86.
- [4] Parepa, S., State of Dynamic Stresses in Zones with Stress Concentrators in Case of High Diameter Drill Pipes, *Petroleum-Gas University of Ploiești Bulletin, Technical Series*, Vol. LXIII, No. 1, 2011, p. 231-236.
- [5] Ponomariov, D., Biderman, V. L., Liharev, K. K., Makușin, V. M., Malinin, N. N., Feodosiev, V. I., *Calculul de rezistență în construcția de mașini*, vol. III, Editura Tehnică, București, 1964.
- [6] Parepa, S., Stand for the study of the non-stationary hydro-mechanical processes in the frame of the electro-hydrostatic driving group from large diameter drilling rigs, *Proceedings of The 11<sup>th</sup> International Symposium of Experimental Stress Analysis and Testing of Materials*, Vol. 3: Modeling and Optimization in the Machines Building Field, Editura ALMA MATER, Bacău, 2006, p. 152-155.