AN ANALYTICAL MODEL FOR STRENGTH CALCULUSOF HIPERSTATIC STRUCTURES

MIRIŢOIU COSMIN[1*], ILINCIOIU DAN[1], JIMENEZ JUAN PABLO[1], ROŞU CĂTĂLIN[1]

[1] *University of Craiova, Faculty of Mechanics, Calea Bucureşti Street, No. 107, Code 200512, Craiova, Romania*

Abstract: In this paper, it is presented an analytical model based on the "forced method" and Mohr-Maxwell theorem, for strength calculus of three dimensional hiperstatic structures under static loadings. The calculus results are parametric for generalizing the solution. In the end, the theoretical aspects are used for a particular case.

Keywords: hiperstatic structures, mechanical strength, mechanical stress, bending moment, torque moment, simplifying hypothesis

1. INTRODUCTION

Many metallic structures, that are designed to sustain different loadings, used in practical engineering are hiperstatic (the unknown parameters given by the reaction forces and closed contours are higher than the equilibrum equations written for a three dimensional body).

In this paper it is made an analytical model for strength calculus of a hiperstatic three dimensional metallic structure, that represents a truck chasis, under static loadings. Other different analytical methods were also presented in [1], [2] and [3] for distinct indeterminate metallic structures This model is based on the "force method" and Mohr-Maxwell theorem, presented in [4], [5] and [6], and the calculus relations are parametric for generalizing the solution. There have been used the next simplifying assumptions: the loadings are considered to be in the bodywork mounting points, the structure material is S235JR, there are considered only the bending and torque effects, the beams have a transversal box with rectangular hollow section. The geometry of the metallic structure and its loading scheme is presented in Figures 1 and 2.

The first step of the strength calculus is to define the stiffness constant ratios using relation (1), where E and G are the material elastic modules, the first index of the constants corresponds to the inertia moment type (axial or polar) and the second index corresponds to the beam's number.

$$k_{h1} = \frac{I_{v1}}{I_{h1}},\ k_{t1} = \frac{E}{G}\cdot\frac{I_{v1}}{I_{p1}},\ k_{v2} = \frac{I_{v1}}{I_{v2}},\ k_{h2} = \frac{I_{v1}}{I_{h2}},\ k_{t1} = \frac{E}{G}\cdot\frac{I_{v1}}{I_{p1}} \tag{1}$$

[*] Corresponding author, email: miritoiucosmin@yahoo.com

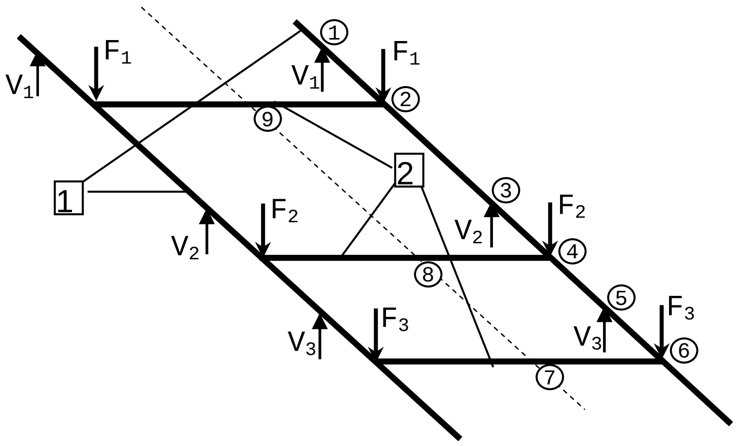


Fig.1.The geometry and loading scheme of the metallic structure: 1- longitudinal beams, 2- transversal beams.

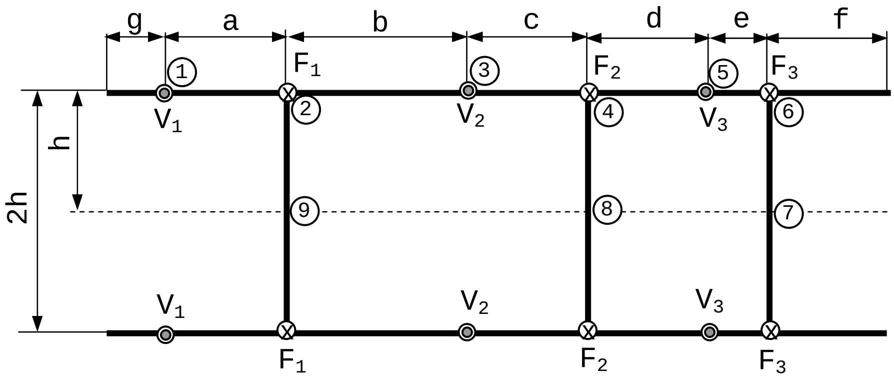


Fig.2. The horizontal plane view of the geometry and loadings.

2. THE BASE AND SECONDARY SYSTEMS SOLVE. THE STRESSES CALCULUS.

From Figures 1 and 2 we can say that the structure is five times statically undetermined. The calculus scheme of the base system is presented in Figure 3.

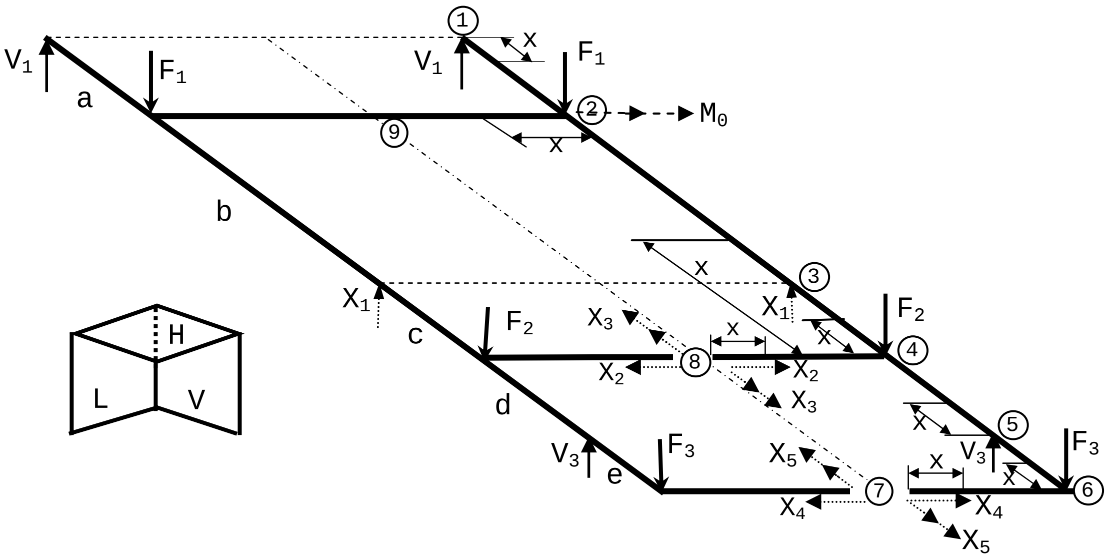


Fig.3. The basic system with real loadings described by all the three planes: horizontal, vertical and lateral.

The base system is obtained by sectioning the two closed contours (we have the unknown parameters $X_2 \ldots X_5$) and by removing the reaction force V_2 (we have the unknown parameter X_1). We will next write the bending and torque moment's equations, defined by the x parameter which has different values depending on the considered gap:

gap 1-2, x϶[0,a] ; gap 2-9, x϶[0,h] ; gap 6-5, x϶[0,e] ; gap 7-6, x϶[0,h] ;

gap 5-4, x϶[0,d] ; gap 4-3, x϶[0,c]; gap 8-4, x϶[0,h] ; gap 3-2, x϶[c,c+b].

$$M_{120v} = \frac{F_1(b+c+d)+F_2 d - F_3 e}{a+b+c+d}\cdot x \, ; \; M_{650v} = -F_3 \cdot x \, ; \qquad (2)$$

$$M_{540v} = \frac{F_1 a \cdot x + F_2(a+b+c)\cdot x - F_3 e\cdot(a+b+c+x)}{a+b+c+d}, \; M_{290t} = 2\cdot F_2 e$$

$$M_{430v} = \frac{F_1(ad+xa)+F_2(cd+bd-xd)-F_3 e\cdot(c+x+2d+a+b)}{a+b+c+d} = M_{320v},$$

The moments equal to zero on the considered gaps, haven't been written in relation 2. In order to write the secondary systems, we ignore the real loadings and we insert a unitary force or moment in the points where the unknown parameters $X_1 \ldots X_5$ exist. Five secondary systems are created.

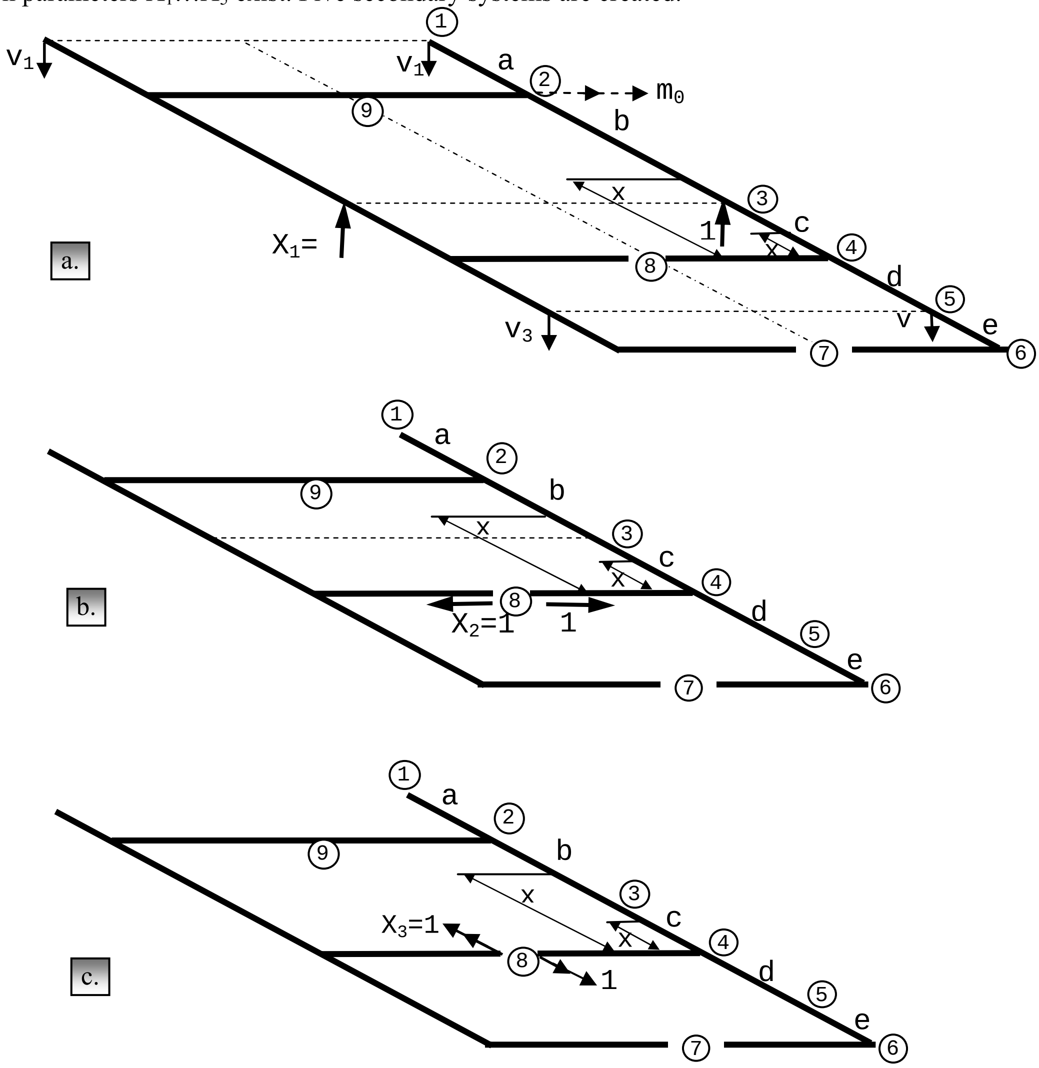
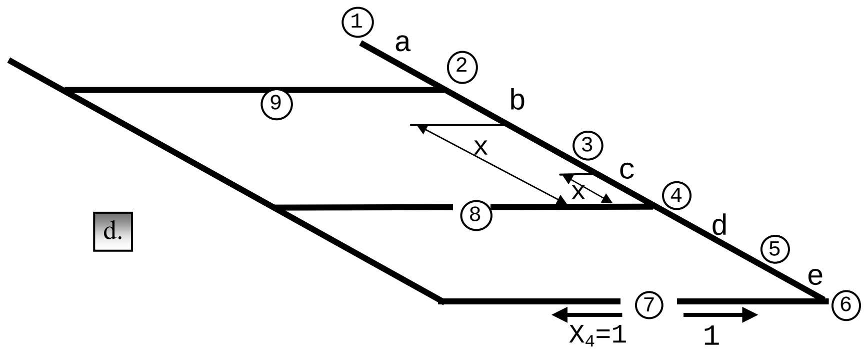


Fig.4. The secondary systems : a. – the first secondary system; b – the second secondary system; c - the third secondary system.



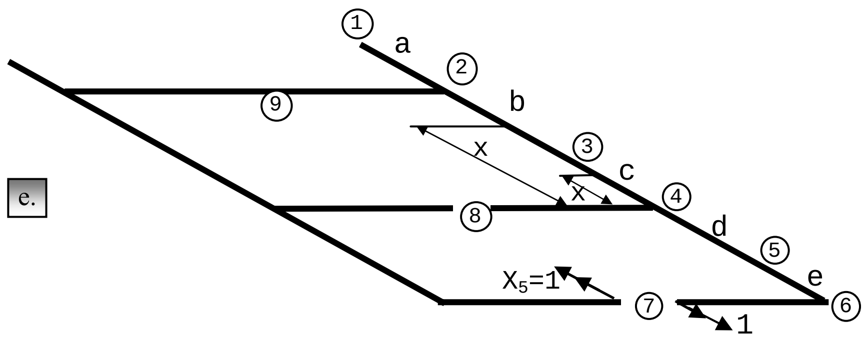


Fig.4. The secondary systems: d – the fourth secondary system; e- the fifth secondary system.

We write the bending and torque moments equations on each gap for every secondary system from Figure 4.

$$m_{121v} = -\frac{c+d}{a+b+c+d}\cdot x\,,\; m_{541v} = -\frac{a+b}{a+b+c+d}\cdot x\,,\; m_{431v} = -\frac{a+b}{a+b+c+d}\cdot(d+x), \quad (3)$$

$$m_{321v} = -\frac{a+b}{a+b+c+d}\cdot(d+x)+(x-c),$$

$$m_{291t} = -\frac{c+d}{a+b+c+d}\cdot a - b + \frac{a+b}{a+b+c+d}\cdot(b+c+d).$$

$$m_{432h} = x\,,\; m_{322h} = x\,,\; m_{292h} = b+c\,,\; m_{843v} = -1,\; m_{433t} = 1,\; m_{323t} = 1,\; m_{293v} = 1,$$

$$m_{654h} = x\,,\; m_{544h} = x\,,\; m_{434h} = e+d+x\,,\; m_{324h} = e+d+x\,,\; m_{294h} = b+c+d+e\,,$$

$$m_{765v} = -1,\; m_{655t} = 1,\; m_{545t} = 1,\; m_{435t} = 1,\; m_{325t} = 1,\; m_{295v} = 1.$$

The moments equal to zero on the considered gaps for all the secondary systems, haven't been written in relation 3. Using the moments relations (2) and (3) and Mohr-Maxwell method, we create the canonical system (4) and its factors a_{ij} are calculated with (5). Using the system (4) we obtain the unknown parameters $X_1..X_5$.

$$\begin{cases} a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + a_{14}X_4 + a_{15}X_5 + a_{10} = 0 \\ \ldots \\ a_{51}X_1 + a_{52}X_2 + a_{53}X_3 + a_{54}X_4 + a_{55}X_5 + a_{50} = 0 \end{cases} \quad (4)$$

$$a_{11} = \int_0^a m_{121v}^2 dx + \int_0^d m_{541v}^2 dx + \int_0^c m_{431v}^2 dx + \int_c^{c+d} m_{321v}^2 dx + k_{t2} \int_0^h m_{291t}^2 dx\,, \tag{5}$$

$$a_{22} = k_{h1} \int_0^{c+b} m_{432h}^2 dx + k_{h2} \int_0^h m_{292h}^2 dx\,,\ a_{24} = k_{h1} \int_0^{c+b} m_{432h} \cdot m_{434h} dx + k_{h2} \int_0^h m_{292h} \cdot m_{294h} dx\,,$$

$$a_{33} = k_{v2} \int_0^h m_{843v}^2 dx + k_{t1} \int_0^{c+b} m_{433t}^2 dx + k_{v2} \int_0^h m_{293v}^2 dx\,,\ a_{35} = k_{t1} \int_0^h m_{433t} \cdot m_{435t} dx + k_{v2} \int_0^h m_{293v} \cdot m_{295v} dx\,,$$

$$a_{44} = k_{h1} \int_0^e m_{654h}^2 dx + k_{h1} \int_0^d m_{544h}^2 dx + k_{h1} \int_0^{c+d} m_{434h}^2 dx + k_{h2} \int_0^h m_{294h}^2 dx\,,$$

$$a_{55} = k_{v2} \int_0^h m_{765v}^2 dx + k_{t1} \int_0^e m_{655t}^2 dx + k_{t1} \int_0^d m_{545t}^2 dx + k_{t1} \int_0^{c+b} m_{435t}^2 dx + k_{v2} \int_0^h m_{295v}^2 dx\,,$$

$$a_{10} = \int_0^a m_{121v} \cdot M_{120v} dx + \int_0^d m_{541v} \cdot M_{540v} dx + \int_0^c m_{431v} \cdot M_{430v} dx + \int_c^{c+b} m_{321v} \cdot M_{320v} dx + k_{t2} \int_0^h m_{291t} \cdot M_{290t} dx$$

The factors that haven't been presented in (5) are zero. Having calculated the initial indeterminate values $X_1 \ldots X_5$, we can directly determine the bending and torque moments using (6), marking with ij the gaps where the moments are being computed.

$$M_{ij(v,h,t)} = M_{ij0} + m_{ij1} X_1 + m_{ij2} X_2 + m_{ij3} X_3 + m_{ij4} X_4 + m_{ij5} X_5 \tag{6}$$

Taking into account only the moments, we can calculate the normal and shear stresses using relation (7), where ij is the gap number, W is the axial strength modulus and W_p is the polar strength modulus.

$$\sigma_{ij} = \frac{M_{ij(v,h)}}{W}\,,\ \tau_{ij} = \frac{M_{ijt}}{W_p} \tag{7}$$

To demonstrate the calculus viability, this modelling will be used for a particular case defined in this way:

F_1= 1300 N, F_2= 3800 N, F_3= 200 N, a= 110 mm, b= 220 mm, c= 70 mm, d= 90 mm, e= 60 mm, f= 150 mm, h= 70 mm, E= $2.1 \cdot 10^5$ MPa, G= $0.8 \cdot 10^5$ MPa, b_1= h_1= 30 mm, g_1= 2 mm, b_2= h_2= 15 mm, g_2= 2mm.

where b_i, h_i, g_i are the geometrical characteristics of the beams section (base, height and wall thickness).

We calculate first the stiffness constant ratios using relation (1).

I_{v1}= $2.157 \cdot 10^4$ mm^4, I_{h1}= $1.13 \cdot 10^4$ mm^4, I_{p1}= $2.209 \cdot 10^4$ mm^4, W_{v1}= $1.438 \cdot 10^3$ mm^3, W_{h1}= $1.113 \cdot 10^3$ mm^3, W_{p1}= $2.016 \cdot 10^3$ mm^3, k_{h1}= 1.938 , k_{t1}= 2.563 , k_{v2}= k_{h2} = 7.192 , k_{t2}= 12.883.

Then we write the moments equations on each gap and we create the canonical system (4) and its factors a_{ij} will be calculated with (5).

$$a_{11} = 1.896 \cdot 10^6\,,\ a_{22} = 5.87 \cdot 10^7\,,\ a_{24} = 9.222 \cdot 10^7\,,\ a_{33} = 1.75 \cdot 10^3\,,\ a_{35} = 1.47 \cdot 10^3\,,$$
$$a_{44} = 1.525 \cdot 10^8\,,\ a_{10} = -5.891 \cdot 10^9$$

Using the system (4) we obtain the unknown parameters $X_1..X_5$ in (8).

$$X_1 = 3106N \, , \; X_1 = ... = X_5 = 0 \tag{8}$$

Using the results from (8) and the relation (6) we can determine the bending and torque moments from each gap (the values are in N·mm).

$$M_{12v} = 716.287 \cdot x \, , \; M_{65v} = -200 \cdot x \, , \; M_{54v} = -12000 + 1277.341 \cdot x \, , \; M_{29t} = 24 \cdot 10^3$$

$$M_{43v} = 102960.698 - 2522.659 \cdot x \, , \; M_{32v} = -114485.364 + 583.713 \cdot x \, .$$

With relation (7) we can determine the normal and shear stresses (the values are in MPa).

$$\sigma_{12v} = 0.498 \cdot x \, , \; \sigma_{65v} = -0.139 \cdot x \, , \; \sigma_{54v} = -8.437 + 0.888 \cdot x \, , \tag{9}$$

$$\sigma_{32v} = -79.631 + 0.406 \cdot x \, , \; \tau_{29} = 35.503$$

After obtaining the stresses values on each gap, we compare the maximum normal and shear stress with the admissible ones in order to see if the structure resists under the chosen loadings.

3. CONCLUSIONS

The hiperstatic structure analyzed in this paper presents a certain simplifying grade, using a number of assumptions. The most important ones are:

- the loadings are static and considered to be in the bodywork mounting points,
- the metallic structure material is S235JR,
- there are considered only the bending and torque effects,
- in the calculus, there have been used the simplifying assumptions from the “strength of materials” (like Bernoulli or Saint-Vennant hypothesis, the material is perfectly elastic, the material is perfectly homogenous and isotropic, the stresses and strains are proportional, etc.),
- the structure elements have been calculated under the “bar schematization ”,
- the beams have a transversal box with rectangular hollow section.

Even if there have been used those simplifying assumptions, there has been obtained a complex model. Its geometry and loading scheme is presented in Figures 1 and 2. The calculus scheme will no longer be valid if the number of the reaction forces is lower or higher than 6 because the degree of static indeterminacy will decrease or increase and another model will be obtained. The same conclusion can be obtained if the number of the closed contours increase or decrease.

The calculus model defined in this paper has big dimensions and can be easily used if only it is inserted in a mathematical calculus software like Matlab, Maple, Mathcad, Mathematica and so on. The authors have tried to simplify the mathematical model in order to be easily understood by anyone who wants to make strength calculus of hiperstatic structures. The calculus relations were given parametrically in order to generalize the used method. The model has a high generality degree and can only be used for the case presented in chapter one. This strength analysis presented in this paper can serve, as a guide, for solving some similar situations. A secondary purpose followed by the authors is the demonstrative character, trying to define the approaching procedure for this king of study.

In order to demonstrate the viability of the presented model, there has been defined a particular case in chapter 2. There have been determined all the parameters from the model and there have also been obtained the bending and torque moment, normal and shear stresses equations. In Figure 5, there will be presented the stresses graphics on the considered gaps.

From Figures 5 and 6 we can extract the following conclusions:

- all the stresses variations are linear
- the only gap subjected to torque is 2-9
- the shear stress is constant
- the maximum normal stress is obtained on the 4-3 gap
- on 1-2 gap, the maximum stress is obtained for x= 110 mm and its value is 54.804 MPa
- on 6-5 gap, the maximum stress is obtained for x= 60 mm and its value is -8.437 MPa
- on 5-4 gap, the maximum stress is obtained for x= 90 mm and its value is 71.615 MPa
- on 4-3 gap, the maximum stress is obtained for x= 0 mm and its value is 71.615 MPa
- on 3-2 gap, the maximum stress is obtained for x= 70 mm and its value is -51.211 MPa
- all the normal stresses occur in the vertical plane
- on the 1-2, 5-4 and 3-2 gaps the stresses graphics are ascending
- on the 6-5 and 4-3 gaps the stresses graphics are descending

As a further research regarding this study, I consider that this analytical model must be validated by an experimental model (for example using strain gages measurements) and by finite element analysis using specialized softwares like Ansys or Abaqus.

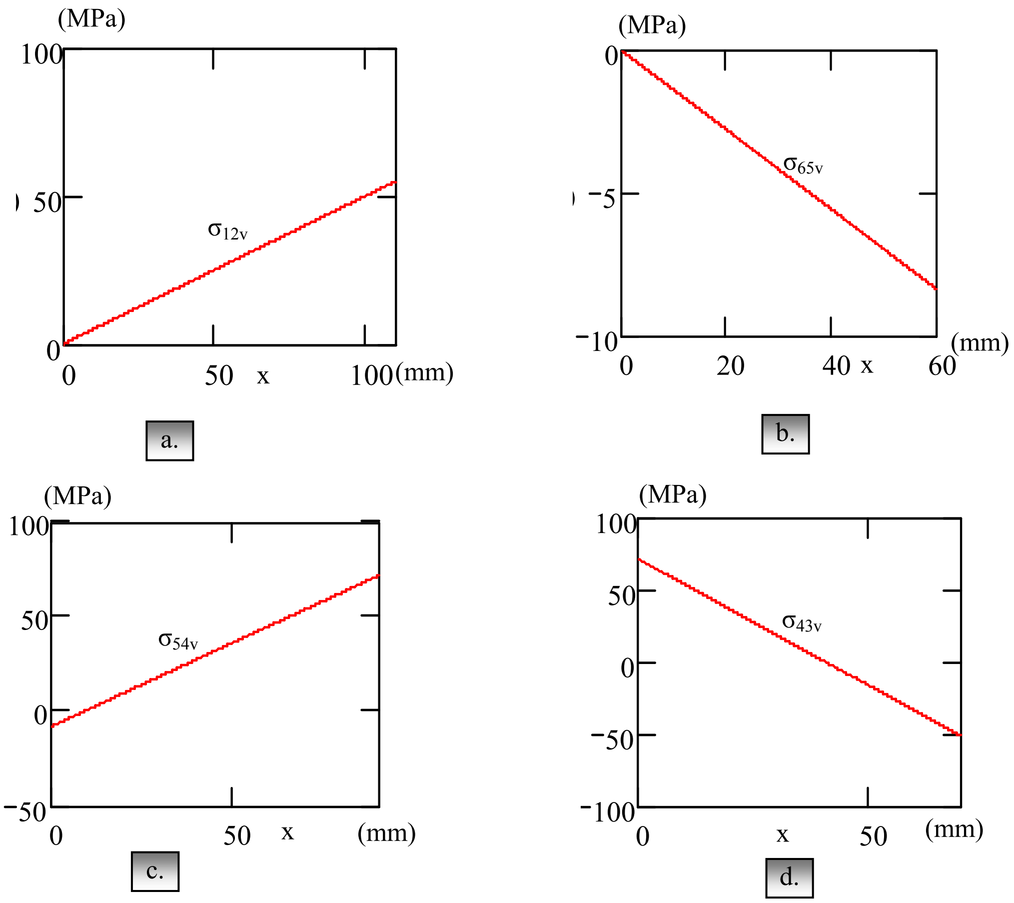


Fig.5. The normal stresses graphics: a- on 1-2 gap, b- on 6-5 gap , c- on 5-4 gap, d- on 4-3 gap.

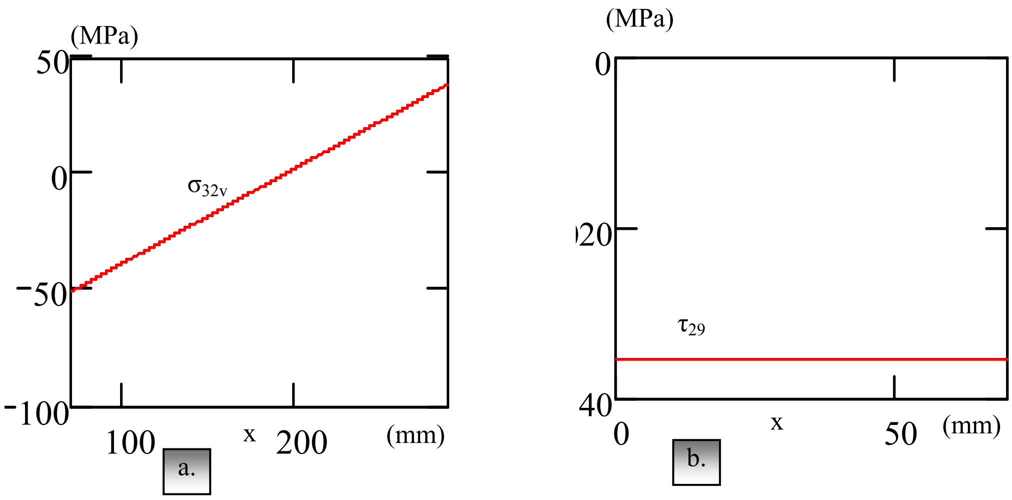


Fig. 6. The normal and shear stresses graphics: a – the normal stress on 3-2 gap, b – the shear stress on 2-9 gap.

ACKNOWLEDGEMENT

This work was partially supported by the strategic grant POSDRU/88/1.5/S/50783 (2009), co-financed by the European Social Fund – Investing in People, within the Sectoral Operational Programme Human Resources Development 2007-2013.

REFERENCES

[1] Ilincioiu , D., Une analise classique de resistance, Annals of the University of Craiova, Mechanical Series, no. 2, 1995, p. 12 – 22.
[2] Ilincioiu, D., Rosca, V., Un efect dinamic asupra structurii principale de rezistenta a unui camion, Annals of the University of Craiova, Mechanical Series, no. 1, 2001, p. 1 – 7.
[3] Ilincioiu, D., Analiza eforturilor din sasiul unui autovehicul, Annals of the University of Craiova, Mechanical Series, no. 1, 2001, p. 7 – 15.
[4] Ilincioiu, D., Rezistenta Materialelor, Editia a-III-a, Ed. ROM TPT, Craiova, 2008.
[5] Ilincioiu, D., Miritoiu, C., Corelation strength-rigidity for a metallic structure, Annals of the University of Craiova, Mechanical Series, no. 1, 2009, p. 3 – 17.
[6] Miritoiu, C., Metode analitice pentru calculul structurilor static nedeterminate, PhD report no. 1, November, 2010.