COMPLEX 3D SHAPES WITH SUPERELLIPSOIDS, SUPERTOROIDS AND CONVEX POLYHEDRONS

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Abstract: In this paper a CAD study for generating of complex 3D shapes with superellipsoids, supertoroids and convex polyhedrons based on computational geometry is presented. The 3D representation was performed using Madsie Freestyle 1.5.3 application. Results from this study also can be adapted and applied in geometric constructions and computer aided design used in engineering and sculpture design.

Keywords: engineering design, sculpture design, superellipsoid, supertoroid, convex polyhedron, implicit surface, CAD.

1. INTRODUCTION

The geometric ideas in computer science, mathematics, engineering, and art have considerable developed. The combination of computer, math, engineering, and art plays an important role in solving a wide variety of problems occurring in computer graphics modeling, computer vision, computer graphics animation and sculpture design [1, 2, 3].

To predict and improve the performance of new designs according to market demand a great diversity of geometry representations has been used for modeling, editing, and rendering of 3D objects [4, 5, 6, 7, 8, 9].

2. SUPERELLIPSOID, SUPERTOROID AND CONVEX POLYHEDRON

2.1. Superquadrics

Superquadrics constitute a class of surfaces which possess a natural parametric and implicit description that were introduced by Barr A. H. in 1981 [10]. The superquadrics are: the superellipsoid, the superhyperboloid of one and two sheets, and the supertoroid [10].

In last decade, superquadrics have received significant attention for object modeling in computer vision and computer graphics [11, 12, 13, 14].

Superquadrics are second-order algebraic surfaces that can be utilized to model only symmetric shapes such as ellipsoids, spheres, cylinders, and parallelepipeds.

A superellipsoid is the spherical product of a pair of two superellipses being defined by the implicit equation [12]:

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$$\left(\left(\frac{x}{a_1}\right)^{2/\varepsilon_2}+\left(\frac{y}{a_2}\right)^{2/\varepsilon_2}\right)^{\varepsilon_2/\varepsilon_1}+\left(\frac{z}{a_3}\right)^{2/\varepsilon_1}=1. \tag{1}$$

A supertoroid is the spherical product of a superellipse and another superellipse with $center_x > a_g$ that is defined by the implicit equation [12]:

$$\left(\left(\left(\frac{x}{a_1}\right)^{2/\varepsilon_2}+\left(\frac{y}{a_2}\right)^{2/\varepsilon_2}\right)^{\varepsilon_2/\varepsilon_1}-a_4\right)^{2/\varepsilon_1}+\left(\frac{z}{a_3}\right)^{2/\varepsilon_1}-1=0. \tag{2}$$

The two exponents are squareness parameters; they are used to pinch, round, and square off portions of the solid shapes, to soften the sharpness of square, and to produce edges and fillets of any arbitrary degree of roundness [10].

This form provides information on the position of a 3D point related to the superquadric surface, which is important for interior/exterior determination [10, 13].
We have an inside-outside function F (x, y, z):

- $F(x, y, z) = 1$ when the point lies on the superquadric surface;
- $F(x, y, z) < 1$ when the point is inside the superquadric surface;
- $F(x, y, z) > 1$ when the point is outside the superquadric surface.

The existence of the inside-outside functions means that superquadrics can be manipulated by means of solid boolean operations, such as union, intersection, and subtraction [10]. The compact shape can be described with a small set of parameters ending up in a large variety of different basic shapes.

Superquadrics can be combined with free-form splines to add the possibility to taper, bend, and twist the model through global deformation. Superquadrics are an easy class of objects to use because they have well defined normal and tangent vectors. Normal vectors are used in intensity calculations during rendering. Both the normal and tangent vectors are used to calculate the curvature of the surface [14].

2.2. Convex polyhedron

A convex polyhedron [15] is a figure composed of finitely many planar polygons so that:
a) it is possible to pass from one polygon to another through polygons having common sides or segments of sides;
b) the entire figure lies on one side of the plane of each constituent polygon.

It is the second condition that defines convexity; the first means that a polyhedron does not split into parts meeting only at vertices or even disjoint from each other.

A convex solid polyhedron is defined as a body bounded by finitely many planar polygons so that it lies on one side of the plane of each of the polygons. The image boundary of a convex solid is a convex polygon.

3. GRAPHICAL REPRESENTATIONS

The Madsie Freestyle 1.5.3 application was used to generate the 3D objects with superellipsoids, supertoroids and convex polyhedrons [16] and the graphical representations are given in Table 1.

The parameters for superellipsoid are:

- Radius X (r_1) - the radius of the ellipsoid along the X-axis;
- Radius Y (r_2) - the radius of the ellipsoid along the Y-axis;
- Radius Z (r_3) - the radius of the ellipsoid along the Z-axis;
- Stacks (n_1) - the number of segments along the Z-axis;
- Slices (n_2) - the number of radial segments around the ellipsoid;
- Stack Exponent (e_1) - the shape of the ellipsoid;

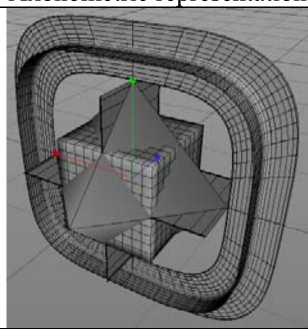
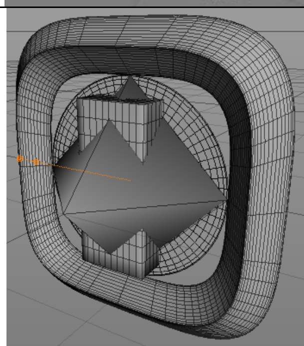
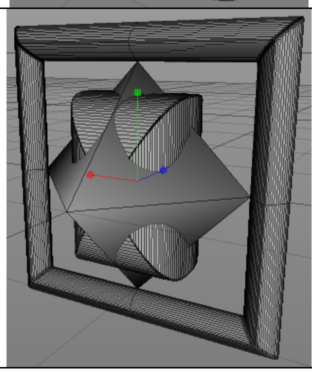
- Slice Exponent (e_2) - the shape of the ellipsoid.

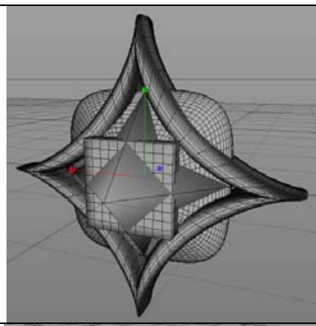
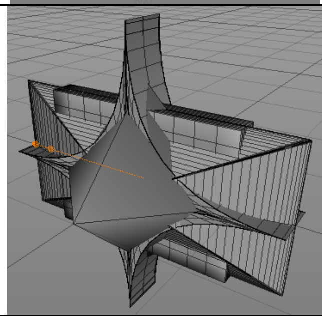
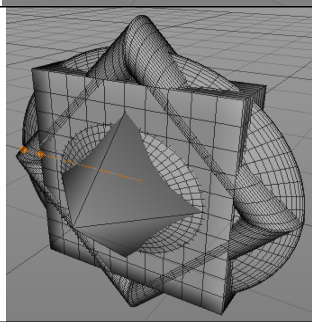
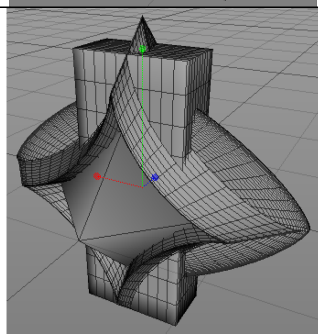
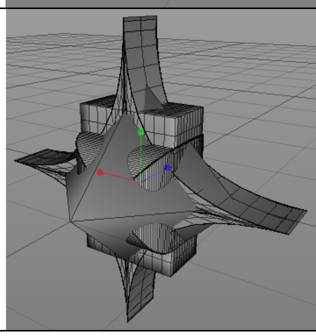
The parameters for supertoroid are:
- Major Radius (r_4) - the radius of the circle that the tube tracks;
- Minor Radius (r_5) - the radius of the tube;
- Major Segments (n_3) - the number of segments along the tube;
- Minor Segments (n_4) - the number of radial segments around the tube;
- Major Exponent (e_3) - the shape of the circle that the tube tracks;
- Minor Exponent (e_4) - the shape of the tube cross section.

The parameters for convex polyhedron are:
- Size X (r_6) - the size of the convex polyhedron along the X-axis;
- Size Y (r_7) - the size of the convex polyhedron along the Y-axis;
- Size Z (r_8) - the size of the convex polyhedron along the Z-axis;
- Segments X (n_5) - the number of segments along the X-axis;
- Segments Y (n_6) - the number of segments along the Y-axis;
- Segments Z (n_7) - the number of segments along the Z-axis.

Because there are some numerical issues in computation with both very small and very large values of the exponents, in this study, for safety, they are chosen in the range of 0.01 to about 8.

Table 1. Graphical representations of 3D complex objects.

No.	Values of parameters	Axonometric representation
1	Superellipsoid: $r_1 = 2$, $r_2 = 2$, $r_3 = 1$, $n_1 = n_2 = 64$, $e_1 = 8$, $e_2 = 0.01$ Supertoroid: $r_4 = 2.5$, $r_5 = 0.5$, $n_3 = n_4 = 64$, $e_3 = 0.5$, $e_4 = 8$ Convex polyhedrons: - cube: $r_6 = 1$, $r_7 = 1$, $r_8 = 1$, $n_5 = n_6 = n_7 = 10$ - octahedron	
2	Superellipsoid: $r_1 = 2$, $r_2 = 2$, $r_3 = 1$, $n_1 = n_2 = 64$, $e_1 = 1$, $e_2 = 8$ Supertoroid: $r_4 = 2.5$, $r_5 = 0.5$, $n_3 = n_4 = 64$, $e_3 = 0.5$, $e_4 = 2$ Convex polyhedrons: - cube: $r_6 = 0.7$, $r_7 = 1.6$, $r_8 = 0.7$, $n_5 = n_6 = n_7 = 10$ - octahedron	
3	Superellipsoid: $r_1 = 1$, $r_2 = 1.5$, $r_3 = 1$, $n_1 = n_2 = 64$, $e_1 = 0.01$, $e_2 = 0.7$ Supertoroid: $r_4 = 2.3$, $r_5 = 0.3$, $n_3 = n_4 = 64$, $e_3 = 0.01$, $e_4 = 1.5$ Convex polyhedrons: - cube: $r_6 = 0.8$, $r_7 = 1.25$, $r_8 = 0.7$, $n_5 = n_6 = n_7 = 10$ - octahedron	

4	Superellipsoid: $r_1 = 2$, $r_2 = 2$, $r_3 = 1$, $n_1 = n_2 = 64$, $e_1 = 1$, $e_2 = 8$ Supertoroid: $r_4 = 2.5$, $r_5 = 0.5$, $n_3 = n_4 = 64$, $e_3 = 0.5$, $e_4 = 2$ Convex polyhedrons: - cube: $r_6 = 0.7$, $r_7 = 1.6$, $r_8 = 0.7$, $n_5 = n_6 = n_7 = 10$ - octahedron	
5	Superellipsoid: $r_1 = 3$, $r_2 = 1.5$, $r_3 = 0.8$, $n_1 = n_2 = 64$, $e_1 = 0.01$, $e_2 = 2$ Supertoroid: $r_4 = 2.5$, $r_5 = 0.5$, $n_3 = n_4 = 64$, $e_3 = 4$, $e_4 = 0.01$ Convex polyhedrons: - cube: $r_6 = 2$, $r_7 = 1.5$, $r_8 = 0.25$, $n_5 = n_6 = n_7 = 10$ - octahedron	
6	Superellipsoid: $r_1 = 3$, $r_2 = 2.5$, $r_3 = 1$, $n_1 = n_2 = 64$, $e_1 = 1$, $e_2 = 2$ Supertoroid: $r_4 = 2.5$, $r_5 = 0.5$, $n_3 = n_4 = 64$, $e_3 = 2$, $e_4 = 1$ Convex polyhedrons: - cube: $r_6 = 2$, $r_7 = 2$, $r_8 = 0.5$, $n_5 = n_6 = n_7 = 10$ - octahedron	
7	Superellipsoid: $r_1 = 3$, $r_2 = 2$, $r_3 = 0.15$, $n_1 = n_2 = 64$, $e_1 = 1.5$, $e_2 = 0.5$ Supertoroid: $r_4 = 2$, $r_5 = 1$, $n_3 = n_4 = 64$, $e_3 = 3$, $e_4 = 2$ Convex polyhedrons: - cube: $r_6 = 1$, $r_7 = 2.5$, $r_8 = 0.5$, $n_5 = n_6 = n_7 = 10$ - octahedron	
8	Superellipsoid: $r_1 = 1$, $r_2 = 1$, $r_3 = 1$, $n_1 = n_2 = 64$, $e_1 = 0.01$, $e_2 = 0.5$ Supertoroid: $r_4 = 2.5$, $r_5 = 0.5$, $n_3 = n_4 = 64$, $e_3 = 4$, $e_4 = 0.01$ Convex polyhedrons: - cube: $r_6 = 1$, $r_7 = 1.5$, $r_8 = 0.5$, $n_5 = n_6 = n_7 = 10$ - octahedron	

4. CONCLUSIONS

This paper presents a CAD method for generation of complex 3D shapes with superellipsoids, supertoroids and convex polyhedrons. The Madsie Freestyle 1.5.3 application helps in obtaining conclusions referring to shape of complex 3D objects, but also facilitate the creation of new analogies in shape design. New generated shapes allow the user to apply a quick series of real-time modeling and to incorporate the resulting solid into other models from engineering and sculpture design.

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REFERENCES

[1] Niţulescu, T., Ţălu, Ş., Applications of descriptive geometry and computer aided design in engineering graphics, Publishing house Risoprint, Cluj-Napoca, Romania, 2001.
[2] Ţălu, Ş., AutoLISP programming language. Theory and applications, Publishing house Risoprint, Cluj-Napoca, Romania, 2001.
[3] Ţălu, Ş., Architectural styles, Publishing house MEGA, Cluj-Napoca, Romania, 2009.
[4] Ţălu, Ş., Descriptive geometry, Publishing house Risoprint, Cluj-Napoca, Romania, 2010.
[5] Ţălu, Ş., Niţulescu, T., The axonometric projection, Publishing house Risoprint, Cluj-Napoca, Romania, 2002.
[6] Ţălu, Ş., Racocea, C., Axonometric representations with applications in technique, Publishing house MEGA, Cluj-Napoca, Romania, 2007.
[7] Racocea, C., Ţălu, Ş., The axonometric representation of technical geometric shapes, Publishing house Napoca Star, Cluj-Napoca, Romania, 2011.
[8] Ţălu, Ş., Ţălu, M., A CAD study on generating of 2D supershapes in different coordinate systems, ANNALS of Faculty of Engineering Hunedoara - International Journal of Engineering, Hunedoara, Tome VIII, Fasc. 3, 2010, p. 201-203.
[9] Ţălu, Ş., Ţălu, M., CAD generating of 3D supershapes in different coordinate systems, ANNALS of Faculty of Engineering Hunedoara - International Journal of Engineering, Hunedoara, Tome VIII, Fasc. 3, 2010, p. 215-219.
[10] Barr, A.H., Superquadrics and angle-preserving transformations. IEEE Computer Graphics and Applications, vol. 1, no. 1, 1981, p. 11-23.
[11] Jaklic, A., Leonardis, A., Solina, F., Segmentation and Recovery of Superquadric. Computational imaging and vision, Kluwer Academic Publishers, Dordrecth, The Netherlands, 2000.
[12] Velho, L., Gomes, J., Figueiredo, L.H., Implicit Objects in Computer Graphics, Springer-Verlag New York, Inc., 2002.
[13] Chevalier, L., Jaillet, F., Baskurt, A., Segmentation and superquadric modeling of 3D objects. Journal of Winter School of Computer Graphics, WSCG'03, 11: 2, 2003, p. 232-239.
[14] Talib, S.B., PhD thesis: Shape recovery from medical image data using extended superquadrics, Buffalo, State University of New York, USA, 2004, p. 21.
[15] Alexandrov, A.D., Convex polyhedra, Springer-Verlag, Berlin, Germany, 2005.
[16] Madsie Freestyle 1.5.3 application by Mads Andersen, Peter Sabroes Gade 17, 3. th, DK - 2450 København SV, Denmark, 2009, at http://www.madsie.com/.